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THE NEUTRON MONITOR TIME SERIES DATA COMMUNICATION WITH THE QUANTUM ALGORITHMS

Annotation

We addressed an elementary task of connecting two *n*-qubit long quantum states by a unitary transformation. The task is similar to the objectives of the Solovay-Kitaev theorem based on the identical assumptions that the number of possible quantum gates (unitary transformations) is uncountable, whereas the number of finite sequences (quantum states) from a finite set is countable. We started with transferring two consecutive time series data sets from the neutron monitor signal into the two quantum state vectors' probability amplitudes. Then we connected these states by the numerically calculated unitary real value transformation matrix. It was sampled from *SO(n)* group by first generating a sequence of random real value entries matrices and second their *QR* factorization into unitary and upper triangular matrices. We ensured the convergence of the process by minimizing the square root of the norma-lized by *n-1* sum of the squared differences between the matrix product of the transformation matrix with the first data set and the second set called σ. The sequence of finer adjustments of the matrix entries was tested to achieve faster and more reliable convergence to the exact unitary transformation. The convergence of the process up to σ=0.008 value was observed with good possibilities for the further improvements. This procedure may be used as a part of the more complex quantum algorithms construction schemes or quantum computation simulation.

Keywords: quantum computations, unitary transformation, state vector, time series, algorithm.

Кілт сөздер: кванттық есептеулер, бірыңғай түрлендіру, күй векторы, уақыттық қатар, алгоритм.

Ключевые слова: квантовые вычисления, унитарное преобразование, вектор состояния, временной ряд, алгоритм.

Introduction. Quantum computations and quantum algorithms have the certain set of limitations on the accepted data format and the quantum gates composition. The data sequence is usually, though not necessarily, transformed into the quantum state vector's probabilities amplitudes and the action of the quantum gates should be described by a unitary transformation which reflects the possibility of the time reversal in the equations describing the quantum phenomena [1]. We wanted to transfer the neutron monitor data in the form of the counts sequences into the simulated quantum computer register and perform unitary operation with them.

Experimental results and Discussions. As a subject of our analysis and numerical calculations we took one minute resolution, 200 counts long, single channel data from the 18NM64 neutron monitor hosted at Tian-Shian high elevation research station, 3340 m above the sea level [2]. The data are spooled and written into the data file every minute so the data piece is 200 minutes long starting from 21/11/2012:16.38.00 UTC (universal time) and ending 21/11/2012:19.57.00 UTC and corrected for atmospheric pressure.

This original signal was divided into two equal parts $S_1(t)$ and $S_2(t)$ and both were assigned to the corresponding state vector $\psi_1=\sum a_i|k_i\rangle$ and $\psi_2=\sum b_j|k_j\rangle$ as the probabilities amplitudes a_i and b_j. Individual neutron counts per minute were normalized before assignment to the complex state vector in such a way that $\sum a_i^2 = \sum b_j^2 = 1$ as expected from the quantum state vector probabilities amplitudes.

The data preprocessing also included the averaging of the both $S_1(t)$ and $S_2(t)$ signals on the 10 minutes basis in order to visually control the process of numerical iterations and get rid of the noise. To observe the general trend of the signal behavior we applied an extra averaging and plotted the outcomes as the dotted line over the plots of original signal, see Fig. 1(b) and (d).

The goal was to connect these two $S_1(t)$ and $S_2(t)$ parts by a unitary transformation matrix Q. That is multiplication of the first state vector by the matrix Q transforms it to the second vector $\psi_2=Q\psi_1$. We have found that the developed process is convergent fast enough (10^{10} iterations and less) though susceptible to

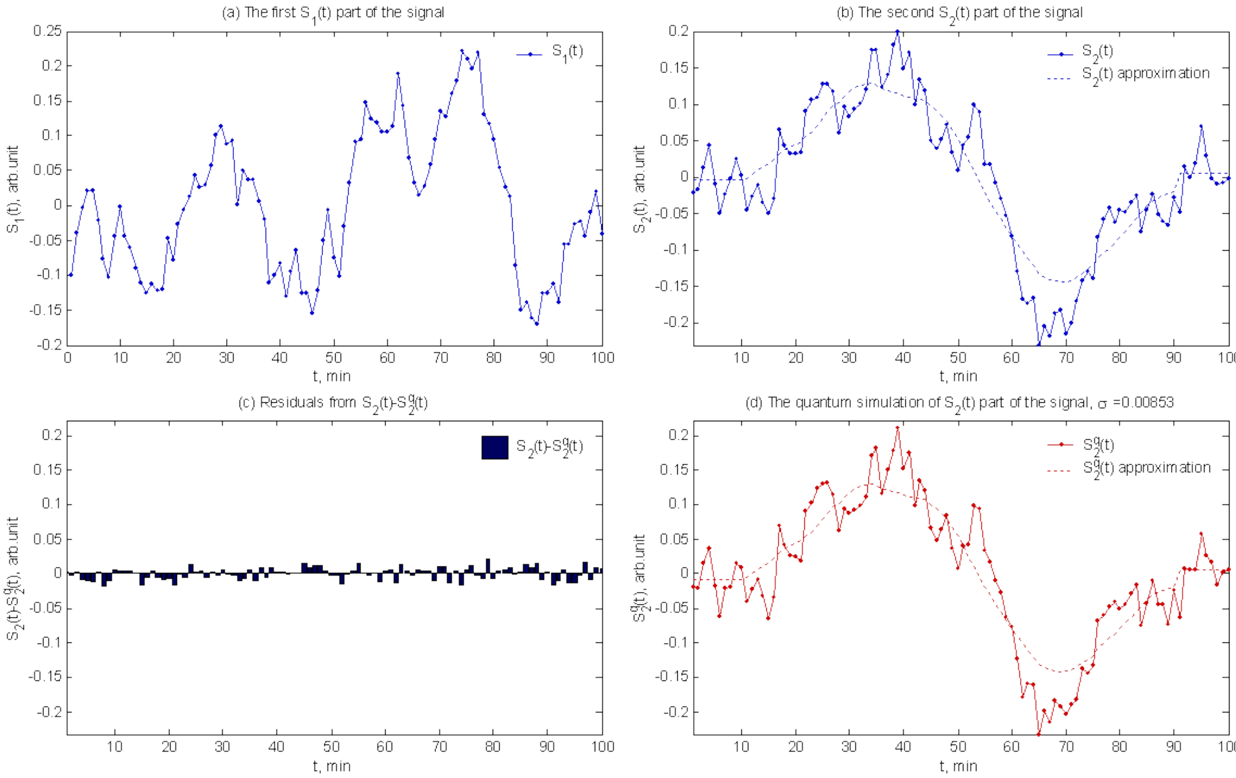

Figure 1 – (a) and (b) The first $S_1(t)$ and the second $S_2(t)$ part of the original signal connected by the unitary transformation matrix Q; (d) The quantum simulation $S_2^q(t)$ of the second part of the signal;

(c) Residuals computed as a difference between $S_2(t)$ and $S_2^q(t)$

the attraction to the local extremum. To control the convergence we calculated the sum of the squared differences between the simulated signal $Q\psi_1$ and its exact values called $\sigma=[\sum(Q\psi_1 - \psi_2)^2/(n-1)]^{1/2}$.

We implemented the procedure outlined in [3]. Using the pseudorandom number generator built into Matlab we generated the n by n matrix A with pseudorandom entries whose values changes from -1 to $+1$. In this way, if n=3, the problem is reduced to the finding of the rotation matrix connecting the two orientations of the vector in 3D space. Taking into account that sequence of numbers generated in Matlab though very long but finite and determined by the seed number we saved the state of the pseudorandom number generator each time the portion of calculation was completed and use it as the seed to continue the next calculations. In such a way we have covered the sequence of non-repeating pseudorandom numbers about n^2 x $10^{10.5}$ long without loss of the debagging data, see Fig.2.

We have used the QR decomposition routine built in Matlab (MATLAB 6.5, The MathWorks Inc., Natick, MA, USA, 2002). QR is the orthogonal/triangular decomposition Matlab function which uses the numerically stable Housholder reflection algorithm and decomposes matrix A

into unitary matrix Q and upper triangular matrix R. These resultant Q matrices are the members of $SO(n)$ group.

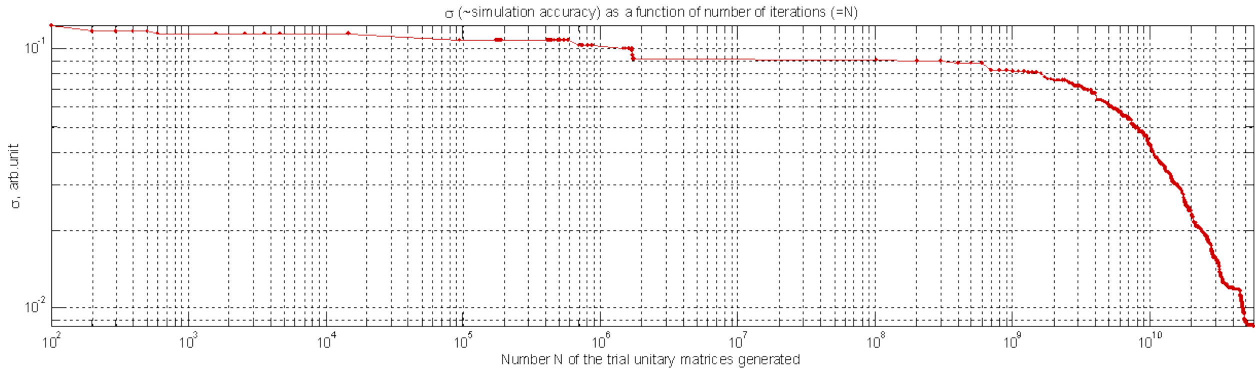

Figure 2 – The σ value as a function of the number of the unitary matrices generated

Random matrix theory accesses the proper probability distribution of the matrices by the Haar measure. The Housholder method though numerically stable produces non-uniform distribution of the eigenvaules of the matrix Q. To correct it and achieve the uniform distribution we followed the procedure outlined in [3]. The piece of Matlab code for this procedure is listed below.

```
A=(rand(signal_length, signal_length)-0.5)*2.0;

[Q,R] = qr(A);

H=sign(R.*eye(signal_length));

Q=Q*H;
```

On Fig.3(a) the distribution of the eigenvalues of the computed Q matrix is plotted. More precisely these are the arguments of the complex eigenvaules belonging to a unit circle. Ideally they should cluster along the $\rho(\theta)={}^{1}/_{2\pi}\sim 0.1592$ value. Some apparent nonuniformity in eigenvalues distribution around zero is visible on the Fig.3(a). It is caused by the fact that we actually plotted the eigenvalues of the matrices already closed enough to the target transformation matrix, not the ones calculated from the very beginning of our code execution. One may consider this plot as an eigenvalues distributions of the computed matrix.

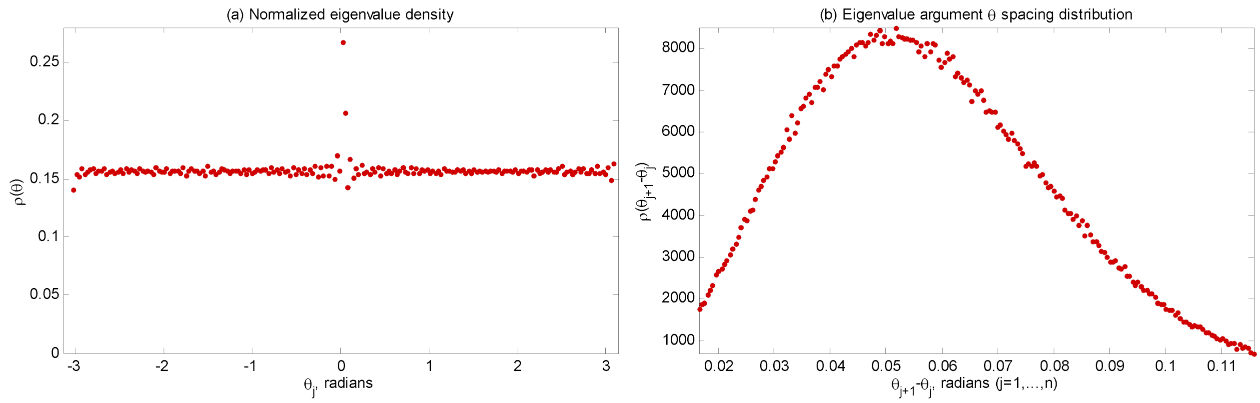

Figure 3 – (a) Normalized eigenvalues density and (b) eignevalues' argument θ spacing distribution.

On Fig.3(b) we can see the corrected distribution of the eigenvaules spacing. One should expect from spacing distribution to have maximum around the $\theta_{j+1}-\theta_j={}^{2\pi}/_{(n-1)} = 0.0628$ where n=100 is the number of eigenvalues and the length of signals $S_1(t)$ and $S_2(t)$. Slight deviation from this number has the same nature as the one described in the previous paragraph.

After we randomly generated and orthogonalized about $10^{9.3}$ matrices we had obtained the pretty good (σ=0.01) approximation of the target matrix and proceeded to the next stage of refinement. That is before the *QR* orthogonalization procedure we were slowly varying a single entry of the matrix *A*, one at the time, testing if this variation after *QR* decomposition is actually improved our approximation. If it did we saved and accepted the new matrix as the next better approximation for the following iterations. At the certain stage the progress exhibited the signs of saturation and we decreased the value of *dA* by a factor of ten. We were able to achieve the σ value of 0.0085 which could me made even less with the subsequent tweaking of the parameters in procedure. Order reversal in which the elements of the matrix *A* are treated turned out to be helpful in the process speedup. The piece of the Matlab code describing these procedures is shown below.

```
factor=100;

for k=N:-1:1

dA=(2*rand-1)/factor;

A(k)=A(k)+dA;

...

end
```

We were also able to observe the numerical effect when the process had collapsed to a local minima when the value *dA* for a single entry was changed too fast and had not being followed by the immediate adjustment of the whole matrix. Fig. 1 shows the results of our numerical simulations. We see very good resemblance between the exact signal $S_2(t)$ and the $S_2^q(t)$ obtained through the quantum simulation algorithm.

Conclusions. It took as about a day on Pentium IV 3.00 GHz computer with 2 Gb of RAM to obtained the result shown on Fig.1. The procedure has proved to be very efficient and straightforward. We are expecting to adopt it for the further time series data analysis. We are able to avoid the explicit construction of the quantum gates. If it is necessary we could derive them from the found unitary transformation matrix using algorithms from the well developed matrix theory [4]. The potential for time reduction and possible applications is substantial.

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Резюме

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НЕЙТРОНДЫҚ МОНИТОРДЫҢ МӘЛІМЕТТЕРІН КВАНТТЫҚ АЛГОРИТМДЕР СИМУЛЯТОРЛАРЫМЕН ӨҢДЕУ

Біз ұзындығы *n* битті құрайтын жүйе күйінің екі кванттық векторының түйісуі туралы жалпы мәселені қарастырдық. Шешімін тапқан бұл мәселе Соловей-Китаевтің теоремасында қарастырылатын, кванттық күйдің санаулы векторлар жиынындағы іріктемелерінің саны шекті бола тұра, мүмкін болатын кванттық вентильдердің немесе унитарлы түрлендірулердің саны шексіз көп болатындығы туралы ұйғарымға негіз-делген мәселеге ұқсас. Нейтрондық монитор сигналынан мәліметтердің екі тізбегін алып, оларды күйдің бастапқы және ақырғы екі кванттық векторының салмақтық коэффициенттеріне түрлендірдік. Одан соң бұл екі кванттық векторды унитарлы Q матрицасының көмегімен, сандық әдістермен таңдап алынған нақты элементтермен байланыстырдық. Табылған матрица $SO(n)$ тобының мүшесі болып табылады, сонымен қатар ол кездейсоқ A матрицасын кездейсоқ нақты мәндермен генерациялау жолымен және оны одан әрі унитарлы және Матлабқа енгізілген QR функциясының көмегімен, жоғары үшбұрышты матрицаға жіктеу арқылы алынған болатын. Қол жеткізіліп отырған процестің жинақтылығы күйдің ақырғы векторының есептелген және нақты мәндерінің арасындағы орташа квадраттық ауытқу σ-ны есептеу арқылы бақыланып отырды. Таңдап алынған A матрицасының мәні мен элементтерінің өзгерісінің жиынтығы унитарлы түрлендіруді анықтау барысын әлдеқайда тездететінін көрсетті. Одан әрі жақсартылу мүмкіндігі бар, $\sigma = 0.008$ мәні алынды. Құрастырылған бағдарламалар және алгоритмдер жиынтығы кванттық есептеулердегі мұнан да күрделі кванттық алгоритмдер мен сұлбалардың құрамдас бөлігі ретінде тиімді түрде қолданыла алады.

Кілт сөздер: кванттық есептеулер, бірыңғай түрлендіру, күй векторы, уақыттық қатар, алгоритм.

Резюме

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ОБРАБОТКА ДАННЫХ НЕЙТРОННОГО МОНИТОРА СИМУЛЯТОРАМИ КВАНТОВЫХ АЛГОРИТМОВ

Мы рассмотрели общую задачу сопряжения двух квантовых векторов состояния системы длинной в n бит унитарным преобразованием. Решенная задача аналогична рассматриваемой в теореме Соловея-Китаева и основывающейся на положении о том, что количество возможных квантовых вентилей или унитарных преобразований бессчетно, в то время как число выборок из счетного множества векторов квантовых состоя-ния конечно. Взяв две последовательности данных из сигнала нейтронного монитора, мы преобразовали их в весовые коэффициенты двух, начального и конечного, квантовых векторов состояния. Затем мы соединили эти два квантовых вектора подобранной численными методами унитарной матрицей Q с действительными элементами. Найденная матрица является представителем группы $SO(n)$ и была получена путем генерации случайной матрицы A с случайными действительными значениями и ее дальнейшем разложением на унитарную и верхнетреугольную матрицу функцией QR встроенной в Матлаб. Достигнутая сходимость процесса контролировалась подсчетом среднеквадратичного отклонения σ между расчетными и точными значениями коэффициентов конечного вектора состояния. Подобранные величина и последовательность вариаций элементов матрицы A показали существенное ускорение процесса нахождения унитарного преоб-разования. Были получены значения σ=0.008 с возможностью ее дальнейшего улучшения. Разработанный набор программ и алгоритмов может быть эффективно использован как часть более сложных квантовых алгоритмов и схем в квантовых вычислениях.

Ключевые слова: квантовые вычисления, унитарное преобразование, вектор состояния, временной ряд, алгоритм.

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